For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Determine the diameter of the circle generated by the polar equation $r = 4\sqrt{2}\sin(\theta + 6\pi)$.
(A) $2\sqrt{2}$ (B) $3\sqrt{2}$ (C) $4\sqrt{2}$ (D) $6\sqrt{2}$ (E) NOTA

2. The average number of freshmen Navya yells at every week is equal to the greatest eigenvalue of this matrix:

$$\begin{bmatrix} 4 & -10 & 10 \\ 0 & 6 & -2 \\ 0 & -2 & 6 \end{bmatrix}.$$

What is the average number of freshmen Navya yells at every week?
(A) 0 (B) 8 (C) 2 (D) 4 (E) NOTA

3. Find the sum of the x-coordinates of the points where the graphs $10x - 4y = -20$ and $25x^2 + 16y^2 = 400$ intersect. Express your answer as an improper fraction.
(A) $-\frac{16}{5}$ (B) $-\frac{12}{5}$ (C) $\frac{10}{5}$ (D) $\frac{12}{5}$ (E) NOTA

4. Find the area enclosed by the parametric equations $x(t) = 5\sin\left(\frac{1}{4}t\right), y(t) = -9\cos\left(\frac{1}{4}t\right)$ for $0 \le t < 8\pi$.
(A) 36π (B) 27π (C) 45π (D) 15π (E) NOTA

5. Evaluate the following expression: $(1 - i)^{2022}$.
(A) -1 (B) $-2^{1011}i$ (C) $-2^{506}i$ (D) $-i^{2022}$ (E) NOTA

6. Velan lost a bet with Jason and owes him \$20. The amount of money Velan has paid Jason thus far is equal to the sum of the sixth roots of unity. How much money does Velan have left to pay?
(A) \$19 (B) \$11 (C) \$14 (D) \$8 (E) NOTA

Use the following information for problems 7 and 8. Tanmay walks on a path that can defined by the expression $y + 7 = -2(x - 3)$. Karthik walks on a path defined by the expression $y = -\frac{1}{3}x + \frac{1}{9}$.

7. Calculate the larger of the two angles, in degrees, formed by the intersection of their two paths.
(A) 105° (B) 120° (C) 135° (D) 150° (E) NOTA

8. Tanmay and Karthik's paths help form a triangle ABC. In triangle ABC, side $AB = 8$, side $BC = 10$, and the measure of angle B is equivalent to the smaller angle formed by the intersection of Karthik's and Tanmay's paths. What is the area of triangle ABC?
(A) $10\sqrt{2} + 10\sqrt{6}$ (B) $20\sqrt{2}$ (C) $20\sqrt{3}$ (D) 20 (E) NOTA

9. Nitish is standing on a cliff that is 400 ft tall when he drops a ball. Each time the ball hits the ground, it bounces up to a height $\frac{3}{5}$ the distance of the previous drop. After the ball has stopped moving, Nitish drops his pet rock Rico. However, Rico does not bounce once he hits the ground. How much greater is the vertical distance traveled by the ball than the vertical distance Rico travels (assume that the objects hit the ground at a height of 0 ft and Rico does not travel any further after his initial drop)?
(A) 600 ft (B) 1000 ft (C) 1200 ft (D) 1600 ft (E) NOTA

10. The equation of the plane that contains the points $P = (-4, -8, 0), Q = (6, 2, 4), R = (5, -2, 3)$ can be expressed in the form $ax + by + cz + d = 0$, where a, b, c, d are integers. Which of the following could not be $|a + b + c + d|$.

(A) 54 (B) 72 (C) 18 (D) 30 (E) NOTA

11. The rational expression $\dfrac{-33x - 59}{x^3 - 19x - 30}$ can be expressed as $\dfrac{A}{x+2} + \dfrac{B}{x+3} + \dfrac{C}{x-5}$. Calculate AB^C.

(A) $\dfrac{1}{625}$ (B) 0 (C) -625 (D) 20 (E) NOTA

12. Given the following equation for a conic $16x^2 - 24xy + 34y^2 - 60 = 0$, we can rotate the axes by an angle θ in order to rewrite it into standard form. Find $\sin(2\theta)$, given $\pi < \theta < \dfrac{3\pi}{2}$.

(A) $\dfrac{2}{5}$ (B) $\dfrac{-3}{5}$ (C) $\dfrac{4}{5}$ (D) $\dfrac{-4}{5}$ (E) NOTA

13. Simplify the following expression: $\dfrac{(\csc x - \cot x)^2 + 1}{(\sec x)(\csc x - \cot x)}$.

(A) $2\sin x$ (B) $2\cos x$ (C) $2\tan x$ (D) $2\sec x$ (E) NOTA

14. Manjari loves to paint. She signs each of her paintings using a special shape represented by the equation $r = 2 - 3\sin(\theta)$. What is the exact shape of Manjari's special signature?

(A) inner loop limacon (B) cardioid (C) lemniscate (D) convex limacon (E) NOTA

15. Find the sine value of the angle formed by the vectors $< 2, 6, 9 >$ and $< 3, 14, 18 >$.

(A) $\dfrac{\sqrt{5635}}{253}$ (B) $\dfrac{252}{253}$ (C) $\dfrac{1}{253}$ (D) $\dfrac{192}{253}$ (E) NOTA

16. Geetika is trying to save her grade in math class and, luckily, her teacher agrees to give her extra credit if she can correctly solve this problem: Find the number of solutions to $4\cos^3(\theta) - 11\cos^2(\theta) - 3\cos(\theta) = 0$ such that $0 \leq \theta < 2\pi$. What answer should Geetika give her teacher?

(A) 3 (B) 6 (C) 4 (D) 7 (E) NOTA

17. At the amusement park, there is a Ferris wheel with a radius of 45 feet whose center is at the point $(0, 50)$. The ferris wheel makes a complete rotation every 8 minutes, moving in the clockwise direction. If Ananya got onto the Ferris Wheel 2 minutes ago at its lowest point, how high above the ground is she 3 minutes from now (assume each unit on the coordinate plane represents 1 foot)? Round your answer to the nearest foot.

(A) 34 ft (B) 38 ft (C) 67 ft (D) 82 ft (E) NOTA

18. In the midst of an epic fight against a demon, Tanjiro's sword travels the path of a unit vector in the same direction as the vector $v = (12, 13, 5)$. Which of the following vectors represents the path Tanjiro's sword follows?

(A) $\left(\dfrac{12}{13}, 1, \dfrac{5}{13}\right)$ (B) $\left(\dfrac{12}{13\sqrt{2}}, \dfrac{1}{\sqrt{2}}, \dfrac{5}{13\sqrt{2}}\right)$ (C) $\left(\dfrac{12}{\sqrt{2}}, \dfrac{13}{\sqrt{2}}, \dfrac{5}{\sqrt{2}}\right)$ (D) (1, 1, 1) (E) NOTA

19. How many of the six trigonometric functions are even functions?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

20. Given complex number $z = a + bi$ such that $3\bar{z} + 3|z| = 6 + 24i$, find $a + b$.

(A) 15 (B) 23 (C) -7 (D) -23 (E) NOTA

21. Calculate $\cot x$ given that $\sin x + \cos x = -\frac{1}{169}$ and $-\pi \leq x \leq 0$.
(A) $-\frac{35}{36}$ (B) $-\frac{68}{69}$ (C) $-\frac{84}{85}$ (D) $-\frac{119}{120}$ (E) NOTA

22. Harry has gotten lost in the xyz plane and is attempting to find his way back to Ron and Hermione. Ron is at the point $(3, -7, 11)$ and Hermione is at the point $(-4, 8, 2)$. Harry is at point $(-5, x, -1)$. For what value of x will Harry be on the same plane as Ron, Hermione, and the origin?
(A) -4 (B) $\frac{11}{50}$ (C) $\frac{7}{25}$ (D) 1 (E) NOTA

23. Find the cross product of $< 6, 3, 4 >$ and $< 5, 1, 2 >$.
(A) $< 30, 3, 8 >$ (B) $< -9, -5, 8 >$ (C) $< 10, 32, 21 >$ (D) $< 2, 8, -9 >$ (E) NOTA

24. For $f(x) = 4\csc(3x) + 7\sin(2x)$, let A = amplitude and P = period of the function. What is $\frac{A}{P}$?
(A) undefined (B) $\frac{2\pi}{3}$ (C) π (D) 2π (E) NOTA

25. The numbers in the hexagonal sequence can be determined using the formula $h_n = \sum_{k=0}^{n-1}(4k+1)$ for $n \geq 1$. The centered hexagonal numbers can be found using the formula $H(n) = 3n^2 - 3n + 1$ for $n \geq 1$. How many positive integers under 100 are both a hexagonal number and a centered hexagonal number?
(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

Use the following information for questions 26 and 27: $z_1 = 4cis\left(\frac{\pi}{3}\right)$ and $z_2 = 24cis\left(\frac{5\pi}{6}\right)$

26. Evaluate $\frac{z_2}{z_1}$.
(A) $\frac{-1}{6}i$ (B) $3 + 3\sqrt{3}i$ (C) $6i$ (D) $-3\sqrt{3} - 3i$ (E) NOTA

27. Evaluate $z_1 \cdot z_2$.
(A) $-16\sqrt{3} + 16i$ (B) $-48\sqrt{3} - 48i$ (C) $-96\sqrt{3} - 96i$ (D) $48\sqrt{3} - 48i$ (E) NOTA

28. Let $\sin(x) + \cos(2x) = 1$. One possible value for $\tan\left(\frac{x}{2}\right)$ can be expressed in the form $A + \sqrt{B}$ where A and B are positive integers. Evaluate $B - A$.
(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

29. Let
$$N = \begin{bmatrix} -4 & -7 & 6 \\ 11 & 3 & -9 \\ -5 & 2 & 8 \end{bmatrix}.$$
Calculate $|3N|$.
(A) 355 (B) 1065 (C) 3195 (D) 9585 (E) NOTA

30. After an unfortunate day at work, Gabby finds that there are limits to her patience. In the spirit of limits, find
$$\lim_{x \to \infty} \frac{(6x^5 - 3x^3 + 2)}{(4x^5 + 7x^2)}$$
.
(A) 1 (B) ∞ (C) $-\infty$ (D) $\frac{3}{2}$ (E) NOTA